Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **May/June 2022**

MARK SCHEME

Maximum Mark: 120

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

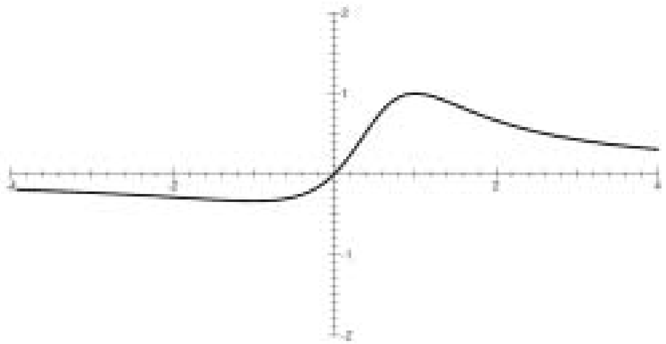
GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	$\dfrac{A}{2n-1}+\dfrac{B}{2n+3}$	**M1**	Correct form and attempt to evaluate A, B
	$A=\frac{1}{4}$ and $B=-\frac{1}{4}$	**A1**	
1(b)	$S\ =\frac{1}{4}\sum_{n=1}^{\infty}\dfrac{1}{2n-1}-\frac{1}{4}\sum_{n=1}^{\infty}\dfrac{1}{2n+3}$ **or** $\frac{1}{4}\sum_{n=1}^{\infty}\left(\dfrac{1}{2n-1}-\dfrac{1}{2n+3}\right)$	**M1**	Use of **(a)**'s result and attempt at difference of two series **or** a series of paired differences
	$=\frac{1}{4}\left(1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+...\right)-\frac{1}{4}\left(\frac{1}{5}+\frac{1}{7}+...\right)$	**M1**	Correct terms identified and attempt at difference (No requirement for examining S_N as $N\to\infty$)
	$=\frac{1}{3}$	**A1**	**CAO** (condone errors in terms $\to 0$)

Question	Answer	Marks	Guidance
2(a)(i)	$y-xy+x^2y=x\Rightarrow yx^2-(y+1)\,x+y=0$	**M1**	Creating a quadratic in x
	$\Delta=(y+1)^2-4y^2$ ($\geqslant 0$ for real solutions)	**M1**	Considering the discriminant
	$\Rightarrow 3y^2-2y-1=(3y+1)(y-1)\leqslant 0\Rightarrow -\frac{1}{3}\leqslant y\leqslant 1$	**A1**	**AG** correctly deduced from $\Delta\geqslant 0$ and visibly correct working

Question	Answer	Marks	Guidance
2(a)(ii)	$y = -\frac{1}{3} \Rightarrow -\frac{1}{3}(x+1)^2 = 0$ giving $x = -1$ $y = 1 \Rightarrow (x-1)^2 = 0$ giving $x = 1$	**M1**	Extreme y-values substituted back (either case)
	TPs at $(-1,\ -\frac{1}{3})$	**A1**	
	and $(1, 1)$	**A1**	Must be clear which x goes with which y *
	Alternative method for question 2(a)(ii)		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1-x+x^2)(1) - x(2x-1)}{(1-x+x^2)^2}$	**M1**	For derivative found, set to zero and eqn. solved
	$= 0$ when $1 - x^2 = 0 \Rightarrow x = 1, -1$ and $y = 1,\ -\frac{1}{3}$	**A1 A1**	One for each correct pair of coordinates*
2(b)		**B1**	HA $y = 0$ (x-axis) noted or clear on sketch
		B1	Continuous curve ('for all x' implied) thro' (0, 0) with TPs in approx. correct places
		B1	Shape

Question	Answer	Marks	Guidance
3(a)(i)	$(a + \mathrm{i}b)^2 = (a^2 - b^2) + \mathrm{i}(2ab)$	**B1**	Correct in $A + \mathrm{i}B$ form (or Re, Im parts identified and used later on)
	$a^2 - b^2 = 28$ and $ab = 48$	**M1**	Re, Im parts equated and solving attempt for a, b
	$\Rightarrow a = 8,\ b = 6$ or $a = -8,\ b = -6$	**A1**	Both

Question	Answer	Marks	Guidance
3(a)(ii)	Similar working gives $c^2 - d^2 = 8$ and $cd = 3$ or $c^2 - d^2 = -8$ and $cd = -3$	**M1**	At least one case considered
	$z = \pm (3 + \mathrm{i})$	**A1**	One correct pair
	or $\pm (1 - 3\mathrm{i})$	**A1**	All four correct solutions and no extras
	Alternative method for question 3(a)(ii)		
	Finding one root and repeatedly multiplying by i	**M1**	
	First correct pair	**A1**	
	All four and no extras	**A1**	
3(b)	Sketch of a circle, centre $28 + 96\mathrm{i}$, passing through O	**B1**	
	$d = \sqrt{28^2 + 96^2} = 100$	**B1**	Note: **not** ± 100

Question	Answer	Marks	Guidance
4(a)	$L = \int \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}\ dx$	**M1**	Attempted **use** of arc-length formula
	$\sqrt{1 + \left(\dfrac{dy}{dx}\right)^2} = \sqrt{1 + (\sinh x)^2} = \cosh x$	**B1**	
	$L = \int_0^1 \cosh x\ dx = [\sinh x]$	**A1**	Correct integration
	$= \sinh 1 = \frac{1}{2}\left(e - \dfrac{1}{e}\right)$	**A1**	**CAO** (any correct form) in terms of e
4(b)(i)	$L \approx \int_0^1 \left(1 + \frac{1}{2}x^2 + \frac{1}{24}x^4\right) dx$	**M1**	Attempted integration of these terms (from Formula Book)
	$= \left[x + \frac{1}{6}x^3 + \frac{1}{120}x^5\right]$	**A1**	Correct integration
	$= 1 + \frac{1}{6} + \frac{1}{120} = \frac{47}{40}$ or 1.175	**A1**	
4(b)(ii)	Truncating the cosh series at any point leaves positive terms unused	**B1**	Or words to this effect

Question	Answer	Marks	Guidance
5(a)	$\mathrm{i}(x) = x$	**B1**	
5(b)	$\mathrm{pq}(x) = \dfrac{1}{1-x}$	**B1**	
	$\mathrm{qp}(x) = 1 - \dfrac{1}{x}$ or $\dfrac{x-1}{x}$	**B1**	
	$\mathrm{pqp}(x) = \dfrac{1}{1-\dfrac{1}{x}} = \dfrac{x}{x-1}$ or $\mathrm{qpq}(x) = 1 - \dfrac{1}{1-x} = \dfrac{-x}{1-x}$	**B1**	
5(c)	{i} and *G*	**B1**	For these two; but B0 for any extra subgroups of order 3 or 4 or …
	{i, p}, {i, q} i.e. $\left\{x,\ \dfrac{1}{x}\right\}$, $\{x,\ 1-x\}$	**B1**	1st B1 for any two correct subgroups of order 2
	{i, pqp} i.e. $\left\{x,\ \dfrac{x}{x-1}\right\}$ or equivalent forms for pqp	**B1**	2nd B1 for third and no extras
	{i, pq, qp} i.e. $\left\{x,\ \dfrac{1}{1-x},\ \dfrac{x-1}{x}\right\}$	**B1**	

Question	Answer	Marks	Guidance
6	$\frac{\mathrm{d}y}{\mathrm{d}x}+\left(-\frac{1}{x}\right)y=\frac{x}{\sqrt{1+x^2}}$	**B1**	Preparatory work (may be implied by following correct working) in this form
	I.F. is $\exp\left\{\int \frac{-1}{x}\,\mathrm{d}x\right\}$	**M1**	Attempt to find the I.F.
	$=\exp(-\ln x)=\frac{1}{x}$	**A1**	Or **B3** for going straight to $\left(\frac{1}{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x}+\left(-\frac{1}{x^2}\right)y=\frac{1}{\sqrt{1+x^2}}$
	Integrating b.s. of $\left(\frac{1}{x}\right)\frac{\mathrm{d}y}{\mathrm{d}x}+\left(-\frac{1}{x^2}\right)y=\frac{1}{\sqrt{1+x^2}}$ w.r.t. x	**M1**	
	$\Rightarrow \frac{y}{x}=\sinh^{-1}x\ (+C)$ or $\frac{y}{x}=\ln\left(x+\sqrt{1+x^2}\right)\ (+C)$	**A1**	LHS
		A1	RHS (condone missing + C here)
	Use of $x=\frac{3}{4}$, $y=3\ln 2$ to evaluate C	**M1**	
	$\Rightarrow y=x\left(\sinh^{-1}x+3\ln 2\right)$	**A1**	Or equivalent log form; must be explicit form and not involving (e.g.) $\sinh^{-1}\frac{3}{4}$ as part of the C

Question	Answer	Marks	Guidance
7(a)	Det(**M**) = $(2k-1)(1-8k)-(1-k)(k-1)$	**M1**	Det(**M**) attempted
	$15k^2-8k=0$	**M1**	Set to zero and solving a quadratic eqn.
	$(k \neq 0) \Rightarrow k = \frac{8}{15}$	**A1**	
7(b)(i)	$(2k-1)^2+(k-1)^2=1$ **or** $(1-k)^2+(1-8k)^2=1$	**M1**	Use of $c^2+s^2=1$ and solving
	$5k^2-6k+1=0$ **or** $65k^2-18k+1=0$		
	$\Rightarrow k = \frac{1}{5}$ or 1 **or** $k = \frac{1}{5}$ or $\frac{1}{13}$ $k=1 \Rightarrow \mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & -7 \end{pmatrix}$ **or** $k = \frac{1}{13} \Rightarrow \mathbf{M} = \begin{pmatrix} -\frac{11}{13} & -\frac{12}{13} \\ \frac{11}{13} & \frac{5}{13} \end{pmatrix}$	**B1**	Visibly rejecting the 2nd case
	so that $k = \frac{1}{5}$	**A1**	
	Alternative method for question 7(b)(i)		
	Alt. $2k-1=1-8k \Rightarrow k=\frac{1}{5}$	**M1 A1** **B1**	**B1** for checking that this gives a valid value for both cos *and* sin (i.e. checking the M_{12} and M_{21} elements also)
7(b)(ii)	$\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$	**M1**	Attempt to identify a correct cos or sin value (condone e.g. $\cos\theta = 0.2$) and calculate an angle
	(anticlockwise) rotation through $\pi - \tan^{-1}\frac{4}{3}$ or 126.9° or 2.21 rads	**A1**	Or clockwise equivalent

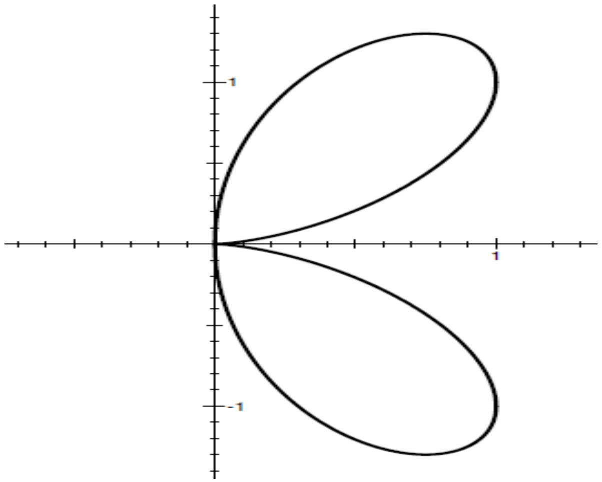
Question	Answer	Marks	Guidance
8(a)	$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$	**M1**	
	$=p^2-2q$	**A1**	
8(b)	$\alpha^2(\beta+\gamma)+\beta^2(\gamma+\alpha)+\gamma^2(\alpha+\beta)=\sum\alpha^2\beta$	**M1**	
	$=\sum\alpha\sum\alpha\beta-3\alpha\beta\gamma$	**M1**	
	$=pq-3r$	**A1**	

Question	Answer	Marks	Guidance
8(c)	$\alpha^3+\beta^3+\gamma^3 = \sum\alpha^3 = \left(\sum\alpha\right)^3 - 3\sum\alpha^2\beta - 6\alpha\beta\gamma$	**M1**	For $\left(\sum\alpha\right)^3 - \ldots$ containing only $\sum\alpha^2\beta$ and $\alpha\beta\gamma$ terms
	$= p^3 - 3(pq-3r) - 6r$	**M1**	Use of previous result for $\sum\alpha^2\beta$
	$= p^3 - 3pq + 3r$	**A1**	
	Alternative method for question 8(c)		
	$\sum\alpha^3 = \sum\alpha^2\sum\alpha - \sum\alpha^2\beta$	**M1**	
	$= (p^2-2q)p - (pq-3r)$	**M1**	Use of both previous results
	$= p^3 - 3pq + 3r$	**A1**	
	Alternative method for question 8(c)		
	Re-arranging given cubic and summing over the 3 roots $\Rightarrow \sum\alpha^3 - p\sum\alpha^2 + q\sum\alpha - r\sum 1 = 0$	**M1**	
	$\Rightarrow \sum\alpha^3 = p(p^2-2q) - q(p) + 3r$	**M1**	Use of **(a)**'s result
	$= p^3 - 3pq + 3r$	**A1**	
	Note that, in **(b)**, $\alpha^2(\beta+\gamma)+\beta^2(\gamma+\alpha)+\gamma^2(\alpha+\beta) = \sum\alpha^2(p-\alpha)$ $= p\sum\alpha^2 - \sum\alpha^3$ so **(c)** can be done this way also		

Question	Answer	Marks	Guidance
9	For $n = 1$, $S_1 = \cos^1\theta\cos(1\times\theta) = \cos^2\theta$ and formula gives $S_1 = \dfrac{\cos^{1+1}\theta\ \ \sin(1\times\theta)}{\sin\theta} = \cos^2\theta$ and result true for $n = 1$	**B1**	Both sides must be clearly demonstrated
	Assume that $S_k = \sum_{r=1}^{k}\cos^k\theta\ \cos k\theta = \dfrac{\cos^{k+1}\theta\ \ \sin k\theta}{\sin\theta}$	**M1**	Clearly stated or explained within the induction 'round up' at the end
	Then $S_{k+1} = \dfrac{\cos^{k+1}\theta\ \ \sin k\theta}{\sin\theta} + \cos^{k+1}\theta\ \ \cos(k+1)\theta$	**M1**	Correct $(k + 1)$th term added to S_k
	$= \dfrac{\cos^{k+1}\theta}{\sin\theta}\left(\sin k\theta + \cos(k+1)\theta\ \ \sin\theta\right)$	**M1**	Sensible factorisation algebra (possibly in the numerator only)
	$= \dfrac{\cos^{k+1}\theta}{\sin\theta}\left(\left[\sin(k+1)\theta\cos\theta - \cos(k+1)\theta\sin\theta\right] + \cos(k+1)\theta\ \ \sin\theta\right)$ *	**M1**	Use of trig. *Addition formula*, with $k\theta = (k+1)\theta - \theta$
	$= \dfrac{\cos^{k+1}\theta}{\sin\theta}\left(\sin(k+1)\theta\cos\theta\right) = \dfrac{\cos^{(k+1)+1}\theta\ \ \sin(k+1)\theta}{\sin\theta}$	**A1**	Clearly demonstrated (or confirmed) of the required form
	Alternative method for question 9 ***(For the final M1 A1 above)***		
	$\sin k\theta + \cos(k+1)\theta\ \ \sin\theta \equiv \sin k\theta + \sin\theta\{\cos k\theta\cos\theta - \sin k\theta\sin\theta\}$ $\equiv\ \sin k\theta + \sin\theta\cos\theta\cos k\theta - \sin k\theta\sin^2\theta$ $\equiv\ \sin k\theta + \sin\theta\cos\theta\cos k\theta - \sin k\theta\left(1-\cos^2\theta\right)$ $\equiv\ \sin\theta\cos\theta\cos k\theta + \sin k\theta\ \ \cos^2\theta\ \equiv \cos\theta\left(\sin\theta\cos k\theta + \sin k\theta\ \ \cos\theta\right)$ $\equiv \cos\theta\ \ \sin(k+1)\theta$	**M1 A1**	
	Proper explanation of the induction logic and validity of the result	**E1**	**Note:** This mark is only gained if the candidate has fully proved the result

Question	Answer	Marks	Guidance
10(a)	$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\frac{1}{2}\left(e^x - e^{-x}\right)}{\frac{1}{2}\left(e^x + e^{-x}\right)}$	**M1**	Exponential forms for sinh and cosh used
	$\dfrac{\frac{1}{2}\left(e^x - e^{-x}\right)}{\frac{1}{2}\left(e^x + e^{-x}\right)} \times \dfrac{e^x}{e^x} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$	**A1**	**AG** legitimately shown (condone the use of $\times e^x$ as a correct indication of method)
10(b)(i)	$\dfrac{e^{4x} - 1}{e^{4x} + 1} - \dfrac{e^{2x} - 1}{e^{2x} + 1} = 0.3$	**B1**	Correct expression for tanh2x
	$\dfrac{u^2 - 1}{u^2 + 1} - \dfrac{u - 1}{u + 1} = 0.3 \quad \Rightarrow (u^2 - 1)(u + 1) - (u^2 + 1)(u - 1) = 0.3(u^2 + 1)(u + 1)$	**M1**	Forming a polynomial in u
	$\Rightarrow 3u^3 - 17u^2 + 23u + 3 = 0$	**A1**	Correct (any non-zero multiple of the standard cubic form)
10(b)(ii)	$\Rightarrow (u - 3)(3u^2 - 8u - 1) = 0$	**M1**	Analytical method for finding all 3 roots
	$\Rightarrow u = 3$ or $u = \dfrac{8 \pm \sqrt{76}}{6}$ or $\dfrac{4 \pm \sqrt{19}}{3}$	**A1**	Correct (and exact)
	$e^{2x} = 3$ or $\dfrac{4 + \sqrt{19}}{3}$	**M1**	Reverting to x's and taking logs (condone extraneous solution at this stage)
	$x = \frac{1}{2}\ln 3$	**A1**	
	$x = \frac{1}{2}\ln\left(\dfrac{4 + \sqrt{19}}{3}\right)$	**A1**	

Question	Answer	Marks	Guidance
11(a)	$\begin{pmatrix}3\\-1\\1\end{pmatrix}\bullet\begin{pmatrix}8\\1\\-3\end{pmatrix}=24-1-3=20$ and $\begin{pmatrix}3\\2\\4\end{pmatrix}\bullet\begin{pmatrix}-1\\1\\1\end{pmatrix}=-3+2+4=3$	**B1**	Verifying that V is in Π_1 and that W is in Π_2 (condone lack of explanatory remarks)
11(b)	$\mathbf{d}=\begin{pmatrix}8\\1\\-3\end{pmatrix}\times\begin{pmatrix}-1\\1\\1\end{pmatrix}=\begin{pmatrix}4\\-5\\9\end{pmatrix}$ or $4\mathbf{i}-5\mathbf{j}+9\mathbf{k}$	**M1**	Method for finding the direction vector
		A1	Correct
	e.g. Eliminating y from $\begin{matrix}8x+y-3z=20\\-x+y+z=3\end{matrix}\Rightarrow 9x-4z=17$	**M1**	Method for finding the p.v. of any point on the line
	Choosing $x=1\Rightarrow z=-2\Rightarrow y=6$ so that $\mathbf{a}=\mathbf{i}+6\mathbf{j}-2\mathbf{k}$	**A1**	Any correct **a**, with integer components ($\mathbf{i}+6\mathbf{j}-2\mathbf{k}$ and $5\mathbf{i}+\mathbf{j}+7\mathbf{k}$ the most likely offerings)
11(c)	Require $1+4\lambda>0$, $6-5\lambda>0$ and $9\lambda-2>0$	**M1**	Noting or working with at least 2 inequalities work
	These give $\lambda>-\frac{1}{4}$, $\lambda<\frac{6}{5}$ and $\lambda>\frac{2}{9}\Rightarrow\frac{2}{9}<\lambda<\frac{6}{5}$	**A1**	For obtaining a final range of λ (**ft** their **a**)
	Only integer available is $\lambda=1\Rightarrow U=(5, 1, 7)$	**A1**	Must have shown uniqueness
11(d)	Vol. $OUVW$ is $\frac{1}{6}\lvert\,\mathbf{u}\,.\,\mathbf{v}\times\mathbf{w}\,\rvert$	**M1**	Correct volume formula used
	$=\frac{1}{6}\begin{pmatrix}5\\1\\7\end{pmatrix}\bullet\begin{pmatrix}-6\\-9\\9\end{pmatrix}$ Note: $\mathbf{u}\times\mathbf{v}=\begin{pmatrix}8\\16\\-8\end{pmatrix}$ and $\mathbf{u}\times\mathbf{w}=\begin{pmatrix}-10\\1\\7\end{pmatrix}$	**M1**	A suitable scalar triple product (or determinant) attempted
	$=\frac{1}{6}(24)=4$	**A1**	

Question	Answer	Marks	Guidance
12(a)	$I_n = \int \sin^{n-1}\theta \;\; \sin\theta \,\mathrm{d}\theta$	M1	Correct splitting and use of 'parts' integration
	$= \left[-\sin^{n-1}\theta \;\; \cos\theta\right] + \int (n-1)\sin^{n-2}\theta \;\cos\theta \times \cos\theta \,\mathrm{d}\theta$	A1	Penalise incorrect sign of definite term here, but not again for the 2nd A mark*
	$= 0 + (n-1) \int \sin^{n-2}\theta \,(1-\sin^2\theta)\;\mathrm{d}\theta$	M1	Method for obtaining sines only in 2nd integral
	$= (n-1)\{I_{n-2} - I_n\} \Rightarrow n\, I_n = (n-1)\, I_{n-2}$	A1	**AG** legitimately obtained (subject to *)
12(b)		B1	Loops in Q_4 and Q_1 and nowhere else
		B1	Symmetry in initial line
		B1	Fully correct
12(c)(i)	Area $= 2 \times \frac{1}{2} \int_0^{\pi/2} 16\sin^4\theta \;\cos^2\theta \;\mathrm{d}\theta$	M1	Allow no 2 or lack of limits $\left(-\frac{1}{2}\pi,\ \frac{1}{2}\pi\right)$ for now
	$= 16 \int_0^{\pi/2} \sin^4\theta \left(1-\sin^2\theta\right)\mathrm{d}\theta = 16\, I_4 - 16\, I_6$	A1	

Question	Answer	Marks	Guidance
12(c)(ii)	$I_0 = \int_0^{\pi/2} 1\,\mathrm{d}\theta = \frac{1}{2}\pi$	B1	Starting term (possibly I_2) correct
	$I_2 = \frac{1}{2}I_0 = \frac{1}{4}\pi \quad I_4 = \frac{3}{4}I_2 = \frac{3}{16}\pi \quad I_6 = \frac{5}{6}I_4 = \frac{5}{32}\pi$	M1	Repeated use of reduction formula (involving at least an attempt at I_6 and I_4)
	Area = $16\left(\frac{3}{16}\pi - \frac{5}{32}\pi\right) = \frac{1}{2}\pi$	A1	**CSO**
12(d)	$r = 4\sin^2\theta\cos\theta = 4\ \frac{y^2}{r^2} \times \frac{x}{r}$	M1	Full substitution for θs attempted
	$\Rightarrow (r^4 =)\ (x^2 + y^2)^2 = 4xy^2$	A1	Correct answer, any form, after r s replaced

Question	Answer	Marks	Guidance
13(a)	$\cos 5\theta = \frac{1}{2} \quad \Rightarrow 5\theta = \frac{1}{3}\pi,\ \frac{5}{3}\pi,\ \frac{7}{3}\pi,\ \frac{11}{3}\pi,\ \frac{13}{3}\pi,\ \ldots$	M1	Considering at least 3 possible angles
	$\Rightarrow \theta = \frac{1}{15}\pi,\ \frac{1}{3}\pi,\ \frac{7}{15}\pi,\ \frac{11}{15}\pi,\ \frac{13}{15}\pi$	A1	All five (and no extras)
13(b)(i)	$z^n = \cos n\theta + \mathrm{i}\sin n\theta$	B1	(by *de Moivre's theorem)*
	$z^{-n} = \cos n\theta - \mathrm{i}\sin n\theta$ gives $z^n + z^{-n} = 2\cos n\theta$	B1	**AG** correctly deduced

Question	Answer	Marks	Guidance
13(b)(ii)	$(2\cos\theta)^5 \quad = (z + z^{-1})^5$	**M1**	Either this or $(2\cos\theta)^3$ attempted
	$= (z^5 + z^{-5}) + 5(z^3 + z^{-3}) + 10(z + z^{-1})$	**A1**	
	$(2\cos\theta)^3 = (z + z^{-1})^3 = (z^3 + z^{-3}) + 3(z + z^{-1})$	**A1**	
	giving $x^5 = 2\cos 5\theta + 5(x^3 - 3x) + 10x$	**M1**	Converting relevant terms to x's
	$\Rightarrow 2\cos 5\theta = x^5 - 5x^3 + 5x$	**A1**	
	Alternative method for question 13(b)(ii)		
	$2\cos 5\theta = (\mathrm{c} + \mathrm{is})^5 + (\mathrm{c} - \mathrm{is})^5$	**M1**	With use of binomial expansion(s)
	$= 2(\mathrm{c}^5 - 10\mathrm{c}^3\mathrm{s}^2 + 5\mathrm{cs}^4)$	**A1**	
	$= 2\mathrm{c}^5 - 20\mathrm{c}^3(1 - \mathrm{c}^2) + 5\mathrm{c}(1 - 2\mathrm{c}^2 + \mathrm{c}^4)$	**M1**	
	$= 32\mathrm{c}^5 - 40\mathrm{c}^3 + 10\mathrm{c}$	**A1**	
	$= (2\mathrm{c})^5 - 5(2\mathrm{c})^3 + 5(2\mathrm{c}) = x^5 - 5x^3 + 5x$	**A1**	
13(b)(iii)	$2\cos 5\theta = 1 \quad \Rightarrow x^5 - 5x^3 + 5x - 1 = 0 \Rightarrow (x - 1)(x^4 + \ldots) = 0$	**M1**	Correct eqn. and attempt to turn quintic into quartic
	$\Rightarrow (x - 1)(x^4 + x^3 - 4x^2 - 4x + 1) = 0$	**A1**	Given 'eqn.' correctly deduced as quartic factor
	$x \neq 1$ ($\theta \neq \frac{1}{3}\pi$) gives $x^4 + x^3 - 4x^2 - 4x + 1 = 0$		
	having roots $x = 2\cos\frac{1}{15}\pi$, $2\cos\frac{7}{15}\pi$, $2\cos\frac{11}{15}\pi$ and $2\cos\frac{13}{15}\pi$	**A1**	

Question	Answer	Marks	Guidance
13(c)	Product of roots is $(2\cos\frac{1}{15}\pi)(2\cos\frac{7}{15}\pi)(2\cos\frac{11}{15}\pi)(2\cos\frac{13}{15}\pi) = 1$	**M1**	
	Using $\cos\theta = \sin\left(\frac{1}{2}\pi - \theta\right)$,	**M1**	
	$\cos\frac{1}{15}\pi = \sin\frac{13}{30}\pi$, $\cos\frac{7}{15}\pi = \sin\frac{1}{30}\pi$, $\cos\frac{11}{15}\pi = -\sin\frac{7}{30}\pi$ and $\cos\frac{13}{15}\pi = -\sin\frac{11}{30}\pi$	**A1**	
	giving $\sin\frac{1}{30}\pi\sin\frac{7}{30}\pi\ \sin\frac{11}{30}\pi\sin\frac{13}{30}\pi = \frac{1}{16}$	**A1**	**AG** legitimately obtained
	Alternative method for question 13(c)		
	Let $E = \sin\frac{1}{30}\pi\sin\frac{7}{30}\pi\ \sin\frac{11}{30}\pi\sin\frac{13}{30}\pi$		
	Using $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$, $\sin\theta = \sqrt{\frac{1}{2} - \frac{1}{2}\cos 2\theta}$	**M1**	(Note: all angles are acute)
	$E = \left(\frac{1}{\sqrt{2}}\right)^4 \sqrt{(1-\cos\frac{1}{15}\pi)(1-\cos\frac{7}{15}\pi)(1-\cos\frac{11}{15}\pi)(1-\cos\frac{13}{15}\pi)}$	**A1**	
	$= \frac{1}{4}\sqrt{1 - \frac{1}{2}\Sigma\alpha + \frac{1}{4}\Sigma\alpha\beta - \frac{1}{8}\Sigma\alpha\beta\gamma + \frac{1}{16}\alpha\beta\gamma\delta}$	**M1**	Using the roots of the eqn. in **(b)(iii)**
	$= \frac{1}{4}\sqrt{1 - \frac{1}{2}(-1) + \frac{1}{4}(-4) - \frac{1}{8}(4) + \frac{1}{16}(1)}$		
	$= \frac{1}{4}\sqrt{1 + \frac{1}{2} + -1 - \frac{1}{2} + \frac{1}{16}} = \frac{1}{4}\sqrt{\frac{1}{16}} = \frac{1}{16}$	**A1**	